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MECHANICS.

153. Proposed by **W. J. GREENSTREET, A. M.**, Editor of The Mathematical Gazette, Stroud, Eng.

An equiangular polygon consisting of equal, freely jointed rods, is hung up from vertex, A. The vertices adjacent to A are connected by a light rod of such length that the polygon is still regular. Find the stress in the rod and the reactions at the vertices.

Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be the point of suspension, O, the centroid of the system, T, the stress on the weightless rod FB, nW, the weight of the rods, $FB=x$, $AB=a$, $AO=y$, $\angle OAB=\angle OBA=\theta$, R=reaction.

Suppose the deformation such that the centroid moves vertically through a small space. Then $Tdx-nWdy=0$.

$BG:AB=\sin\theta:1$; $\therefore x=2a\sin\theta$.

$AO:AB=\sin\theta:\sin(\pi-2\theta)$; $\therefore y=a\sin\theta/\sin(\pi-2\theta)$.

$\therefore y=a\sin\theta/\sin2\theta=\frac{1}{2}a\sec\theta$.

$\therefore dx=2a\cos\theta d\theta$, $dy=\frac{1}{2}a\sec\theta\tan\theta d\theta$.

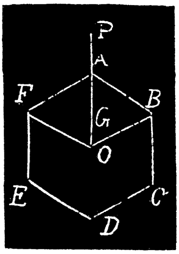
$\therefore 2aT\cos\theta d\theta=\frac{1}{2}(nWa)\sec\theta\tan\theta d\theta$.

$\therefore T=\frac{1}{4}nW\sec^2\theta\tan\theta$. R acts along BO, $\angle ABG=\frac{1}{2}\pi-\theta$.

$\therefore \angle OBG=\angle OFG=2\theta-\frac{1}{2}\pi$, $\angle BOF=2\pi-4\theta$.

$\therefore R:T=\sin(2\theta-\frac{1}{2}\pi):\sin(2\pi-4\theta)$; $R:T=\cos2\theta:\sin4\theta$.

$\therefore R=T\cos2\theta/\sin4\theta=T/2\sin2\theta=T/4\sin\theta\cos\theta$. $\therefore R=\frac{1}{16}nW\sec^4\theta$.



Also solved by *G. W. GREENWOOD.*

154. Proposed by **M. E. GRABER**, Graduate Student, Heidelberg University, Tiffin, Ohio.

Find the form of the curve in a vertical plane, such that a heavy bar resting on its concave side and on a peg at a given point, (the origin), may be at rest at all positions.

Solution by **G. W. GREENWOOD, A. B.**, Professor of Mathematics, McKendree College, Lebanon, Ill.; and **E. L. SHERWOOD**, Professor of Mathematics, Shady Side Academy, Pittsburg, Pa.

Let the bar be homogeneous and of length $2b$. Assume the peg and the curve to be smooth. Take the peg as origin, the horizontal line through it in the plane as initial line, and measure θ downwards.

The coördinates of the lower extremity being (r, θ), those of the center of the rod are $(r-b, \theta)$. Since in all positions the rod is in equilibrium, the center is at a constant distance from the initial line. Hence $(r-b)\sin\theta=a$, is the curve traced by one extremity.

Also solved by *G. B. M. ZERR.*

DIOPHANTINE ANALYSIS.

107. Proposed by **L. C. WALKER, A. M.**, Graduate Student, Leland Stanford Jr. University, Cal.

Required the least three positive integral numbers such that the sum of all three of them and the sum of every two of them shall be a square number.

II. Solution by CHARLES C. CROSS, Whaleyville, Va.

Consider the problem in its most general sense: Required the least n positive integral numbers such that the sum of all of them, and the sum of every $n-1$ of them, shall be squares.

Let x_1, x_2, x_3, ..., x_n be the numbers; then

$$x_1+x_2+x_3+\ldots\ldots+x_n=a^2\,(\text{say})\ldots\ldots(1);$$

$$x_1+x_2+x_3+\ldots\ldots+x_{n-1}=a_1^2\,(\text{say})\ldots\ldots(2);$$

$$x_1+x_2+x_3+\ldots\ldots+x_{n-2}=a_2^2\,(\text{say})\ldots\ldots(3);$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$\ldots\ldots+x_{n-2}+x_{n-1}+x_n=a^2{}_n(\text{say})\ldots\ldots(n).$$

From (2), (3), ..., (n) we find

$$x_1=\frac{1}{n-1}[a_1^2+a_2^2+a_3^2\ldots\ldots-(n-2)a^2{}_n]\ldots\ldots(4).$$

$$x_2=\frac{1}{n-1}[a_1^2+a_2^2-(n-2)a_3^2\ldots\ldots+a^2{}_n]\ldots\ldots(5).$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$x_n=\frac{1}{n-1}[-(n-2)a_1^2+a_2^2+a_3^2\ldots\ldots+a^2{}_n]\ldots\ldots(6),$$

where x_1, x_2, ..., x_n and a are connected by the relation

$$a_1^2+a_2^2+a_3^2+\ldots\ldots+a^2{}_n=(n-1)a^2\ldots\ldots(7).$$

In (7), let $a_2=a-m_1$; $a_3=a-m_2$; $a_4=a-m_3$; ..... $a_n=a-m_{n-1}$; whence

$$a=\frac{a_1^2+m_1^2+m_2^2+\ldots\ldots+m^2{}_{n-1}}{2[m_1+m_2+\ldots\ldots+m_{n-1}]}.$$

From which we readily find,

$$a_2=\{a_1^2-m_1^2+m_2^2+\ldots\ldots+m^2{}_{n-1}-2[m_1m_2+m_1m_3+\ldots\ldots+m_1m_{n-1}]\}$$
$$\div 2[m_1+m_2+\ldots\ldots+m_{n-1}],$$

$$a_3=\{a_1^2+m_1^2-m_2^2+\ldots\ldots+m^2{}_{n-1}-2[m_1m_2+m_2m_3+\ldots\ldots+m_2m_{n-1}]\}$$
$$\div 2[m_1+m_2+\ldots\ldots+m_{n-1}].$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$a_n=\{a_1^2+m_1^2+m_2^2+\ldots\ldots-m^2{}_{n-1}-2[m_1m_{n-1}+m_2m_{n-1}+\ldots\ldots+m_{n-2}m_{n-1}]\}$$
$$\div 2[m_1+m_2+\ldots\ldots+m_{n-1}].$$

These values in (7) give, after reducing,

$$4[m_1+m_2+\dots+m_{n-1}]^2a_1^2+\{a_1^2-m_1^2+m_2^2+\dots+m^2{}_{n-1}-2[m_1m_2+m_1m_3+\dots+m_1m_{n-1}]\}^2+\{a_1^2+m_1^2-m_2^2+\dots+m^2{}_{n-1}-2[m_1m_2+m_2m_3+\dots+m_2m_{n-1}]\}^2+\dots+\{a_1^2+m_1^2+m_2^2+\dots-m^2{}_{n-1}-2[m_1m_{n-1}+m_2m_{n-1}+\dots+m_{n-2}m_{n-1}]\}^2=[n-1][a_1^2+m_1^2+m_2^2+\dots+m^2{}_{n-1}]^2\dots(8).$$

In (8), let a_1^2, $a_2^2\dots a^2{}_n$ represent the first, second.....and nth expressions of the left hand member and a that of the right.

These values of a_1^2, $a_2^2\dots a^2{}_n$ in (4), (5).....(6) give the corresponding values of x_1, $x_2\dots x_n$.

In the problem, $n=3$; whence

$$a^2=[a_1^2+m_1^2+m_2^2]^2,\ a_1^2=4[m_1+m_2]^2a_1^2;$$
$$a_2^2=[a_1^2+m_2^2-m_1^2-2m_1m_2]^2;\text{ and}$$
$$a_3^2=[a_1^2+m_1^2-m_2^2-2m_1m_2]^2.$$

These values of a_1^2, a_2^2 and a_3^2 in (4), (5).....and (6) give

$$x_1=4[a_1^2m_2^2+a_1^2m_1m_2-m_1m_2^3+m_1^3m_2];$$
$$x_2=4[a_1^2m_1^2+a_1^2m_1m_2+m_1m_2^3-m_1^3m_2];\text{ and}$$
$$x_3=a_1^4+m_1^4+m_2^4+2m_1^2m_2^2-2a_1^2m_1^2-2a_1^2m_2^2-8a_1^2m_1m_2.$$

Let $a_1=2$, $m_1=3$, and $m_2=4$; then $x_1=112$, $x_2=672$, and $x_3=57$.
$\therefore x_1+x_2+x_3=29^2$; $x_1+x_2=28^2$; $x_1+x_3=13^2$; and $x_2+x_3=27^2$.

An excellent solution was also received from *A. H. BELL*.

109. Proposed by **HARRY S. VANDIVER**, Bala, Pa.

If $m+n+1$ is a prime integer, show that $m!\times n!-(-1)^{\frac{1}{2}(3m-n)}$ is divisible by $m+n+1$. For instance, $6!\times4!-(-1)^7$ is divisible by 11.

Correction and Solution by **DR. L. E. DICKSON**. The University of Chicago.

In the formula proposed, the exponent of -1 is not correct. Thus, for $m=5$, $n=5$, the number $5!\times5!-(-1)^5$ is not divisible by 11, whereas $5!\times5!-(-1)^4$ is divisible by 11. The correct formula may be stated thus:

$$m!\times(p-1-m)!\equiv(-1)^{m-1}\ (\text{mod}\,p),$$

where p is any odd prime and m any integer $<p$.

The proof follows from the congruences

$$(p-1)!\equiv-1(\text{mod}\,p),\qquad \frac{(p-1)!}{m!(p-1-m)!}\equiv(-1)^m\ (\text{mod}\,p),$$